

Comment on the “ θ -term renormalization in the (2+1)-dimensional CP^{N-1} model with θ term”

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Abstract

It is found that the recently published first coefficient of nonzero β -function for the Chern-Simons term in the $1/N$ expansion of the CP^{N-1} model is untrue numerically. The correct result is given. The main conclusions of the paper [1] are not changed.

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In a recent paper [1], S.H. Park investigated the $1/N$ expansion in the (2+1)-dimensional CP^{N-1} model with a Chern-Simons (or θ -) term and showed that the θ -term does acquire infinite radiative corrections in the first order of $1/N$. We repeated these calculations and found the complete agreement with these conclusions but a different value of the β -function of the θ charge:

$$\beta(\theta) = \frac{320}{9\pi^2} \frac{\theta^3}{(1+\theta^2)^2} \frac{1}{N}$$

The disagreement between our and S.H. Park's results is in the calculation of diagrams (5a)-(5e) from [1]. *Firstly*, the diagrams (5d) and (5e) can be represented, respectively, as (5b) and (5c) but with the reverse orientation of arrows in one of two circles. Since every circle has only two $\bar{n}A_\mu n$ vertices containing momentum and one $\bar{n}\alpha n$ vertex, the contribution of diagram (5b) is independent of orientation of arrows and will not change if arrows are reversed in one of two circles. Hence, the contributions from (5b) and (5d), for example, do not cancel each other like it was proposed in [1] but they are summed. The infinite parts of four diagrams (5b) - (5e) coincide and equal

$$-\mu^{-2\epsilon} \frac{1}{18\pi^2} \frac{\theta}{1+\theta^2} \epsilon^{\mu\rho\nu} p^\rho \frac{1}{\epsilon} \frac{1}{N}.$$

Secondly, our calculation of the contribution of diagram (5a) yields a result which is two times as small as Park's one. We assume that the reason for this may be a wrong double count of orientation of arrows (change of orientation of ones does not result in a new diagram). And *in the third place*, our last notation is that for the correspondence between the Lagrangian and Feynman rules, coefficient of the θ -term in the Lagrangian must be two times as large as one written by author. Comparing this Lagrangian with one in the author's previous paper [2], we confirm our assumption.

We calculated the singular parts of the contributions of diagrams (5a), (5b) and (5d) from [1] by the following way. The leading (at large p^2 , where p is the external momentum) contribution of every diagram, which leads to the renormalization of θ , has the form $A\epsilon^{\mu\rho\nu} p^\rho / (p^2)^l \epsilon$, where l is the loop number and A is the required coefficient. After the differentiation with respect to p^σ , the singular part of every diagram³ does not depend on momentum p and may be found by Vladimirov's method [3], where external momentum is put equal to zero (in principle, there is a necessity of introducing also some masses to preserve a solution from infrared singularities but this is not the case).

The whole sum of the infinite parts of all diagrams (5a) - (5e) are

$$-\mu^{-2\epsilon} \frac{5}{9\pi^2} \frac{\theta}{1+\theta^2} \epsilon^{\mu\rho\nu} p^\rho \frac{1}{\epsilon} \frac{1}{N}.$$

To cancel this infinity we must add to the Lagrangian the corresponding counter-term, which results in the following expression for the bare charge:

³more exactly kR' of diagram but in our case it coincides with the singular part because there are only $1/\epsilon$ terms

$$\theta_0 = \mu^{-2\epsilon} \left(\theta + \frac{80}{9\pi^2} \frac{\theta}{1+\theta^2} \frac{1}{\epsilon} \frac{1}{N} \right).$$

From here we can derive the β -function that is written above.

To conclude, note that the main result of [1] about the occurrence of infinite renormalization of the θ -term in the case of the $1/N$ expansion does not lose its importance. The function $\beta(\theta)$ is nonzero and all main conclusions of the paper [1] are not the subject of a critical review in our comment.

Note only that the results of [1] are in contradiction with the usual weak-coupling expansion where the non-renormalization theorem was established (see [4]). Technically, the appearance of the nonzero β -function in the $1/N$ expansion is quite clear. There is $1/N$ -resummation of the photon propagator. Another (half-integer) power of p^2 is obtained in the ultraviolet range and ultraviolet singularities start to appear already in the leading order of the $1/N$ expansion. Will higher order contributions lead to the permanent saturation of this effect? It is an open question.

Perhaps, the calculation of the next ($1/N^2$) correction might help to illuminate this process. However, usually the calculation of higher order contributions in the framework of the $1/N$ expansion is not a very simple problem.

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References

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